# A stylized model for energy, population, the economy and the environment

## Master Thesis defense

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# Outline

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# A scenario approach that set GDP and population

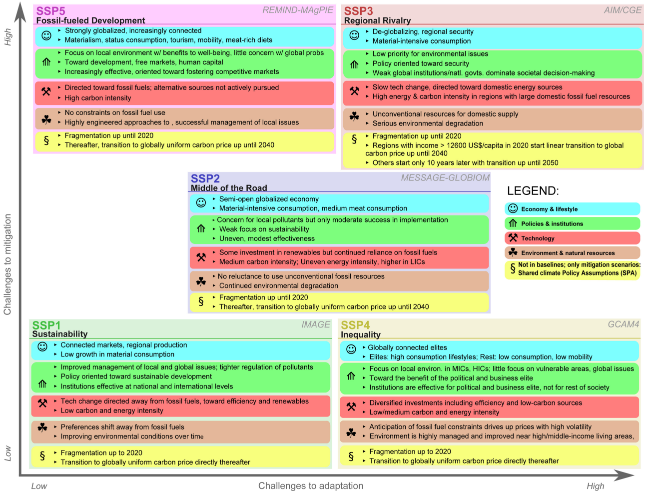



Figure – SSP Scenarii definition - Source : [Bauer et al., 2017]

# Kaya identity : a simple accounting equation

$$CO_{2,t} = \frac{CO_{2,t}}{E_t}\frac{E_t}{Y_t}\frac{Y_t}{Population_t}Population_t$$

# Kaya identity : a graphical representation

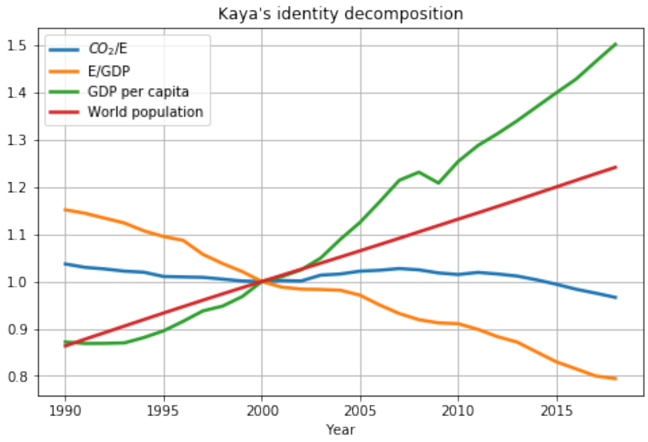



Figure – Kaya decomposition 1990-2018 - Sources : British Petroleum ([British Petroleum, 2019]), World Bank

# $\frac{CO_{2,t}}{E_t}$ : a matter of energy mix

Emission intensity depends on the type of fuel :

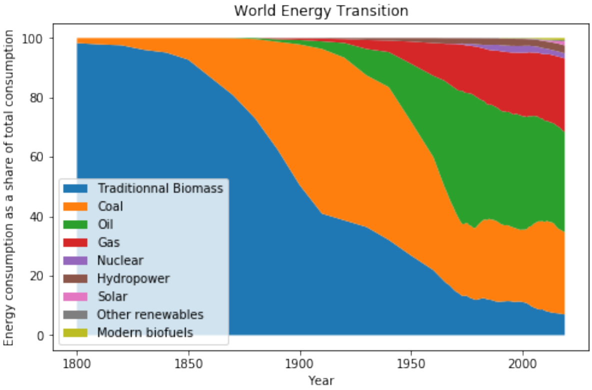
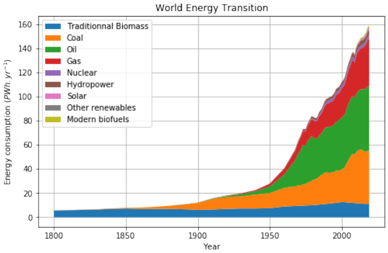



Figure – Qualitative energy transition 1800-2017 - Sources : [Smil, 2016], [British Petroleum, 2019]

# $\frac{E_t}{Y_t}$ : a long term interplay between energy and GDP



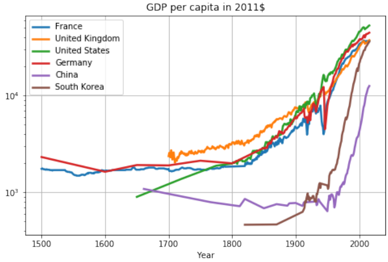




Figure – Quantitative energy transition 1800-2017 and GDP per capita 1500-2016 - Sources : [Smil, 2016], [British Petroleum, 2019], [Bolt et al., 2018]

# Inspiration for the modelling framework

Key ideas and insightful papers :

- The long term interplay between growth and population : Unified growth theory and in particular [Galor and Mountford, 2008]
- The need of primary energy as the fuel of economic production : Ecological economics and in particular [Stern and Kander, 2012]
- The feedback of carbon emissions on societies : Climate economics and in particular the DICE model ([Nordhaus, 2017])

# Outline

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# Overview

- An OLG model à la "unified growth theory" showing endogenous fertility based on a quality-quantity trade-off.
- Agents live on three three periods. For an agent in the age group $t$ : childhood $(t-1)$ during which they are dependent on their parents, who make their own educational choices ; active period/parenthood $(t)$ during which agents work and have children and retirement period $(t+1)$ during which agents benefit from their savings.
- Two types of agents (skilled $s$ and unskilled $u$) : the difference comes from the time of education as well as the expected hourly wage $w_t^s$ and $w_t^u$, there appears to be a competition between the number and type of agents that is the driving force of the demographic process in our model.

# Overview

- Agents can consume two types of goods, basic necessities from agriculture ($c^a_{t,t}$ and $c^a_{t,t+1}$) and manufactured goods (($c^m_{t,}$ and $c^m_{t,t+1}$). During their active period, agents face a subsistence constraint on the consumption of the agricultural good $\tilde{c}_a$, during the childhood period, agents consume a fixed share of $\tilde{c}_a$.
- Agents of type $i$ obtain income from their working time (which depends on their choices in terms of fertility) $w^i_t$ as well as income from renting the land $\frac{r_{L,t}X}{N_t}$ for agriculture as well as for traditional biomass.
- One (very approximate and strong) assumption is that agents do not consume energy. Perhaps including energy in the utility function may be relevant to take into account heating and transport, which are major energy consumers and emitters of greenhouse gases.

# Utility function

An agent of type $i$ has the following utility function :

$$U_t = u^c_{i,t,t} + \delta u^c_{i,t,t+1} + \nu ln(n^u_{i,t} w^u_{t+1} + n^s_{i,t} w^s_{t+1})$$

Where $u^c_{i,t,T} = \theta ln(c^a_{i,t,T}) + (1-\theta) ln(c^m_{i,t,T})$ and $T = t, t+1$ with the following constraints :

$$\begin{aligned} p^m_t s_t + p^a_t \left(c^a_{i,t,t} + (n^u_{i,t} + \gamma^s n^s_{i,t})\epsilon_a \tilde{c}_a\right) + p^m_t c^m_{i,t,t} &\leq w^i_t \left(1 - \tau(n^u_{i,t} + \gamma^s n^s_{i,t} \tau^s)\right) + \frac{r_{L,t} X}{N_t} \\ p^a_{t+1} c^a_{i,t,t+1} + p^m_{t+1} c^m_{i,t,t+1} &\leq s_t r_{K,t+1} \\ \tilde{c}_a &\leq c^a_{i,t,t} \end{aligned}$$

Thus, there a different budget allocation whether the subsistence constraint is binding or not.

# Budget allocation

If the subsistence constraint is binding (i.e. $\frac{\theta}{1+\delta+\nu}\frac{I}{p_t^a} \leq \tilde{c}_a$) :

$$\begin{cases} c^a_{i,t,t} &= \tilde{c}_a \\ c^m_{i,t,t} &= \frac{1-\theta}{1-\theta+\delta+\nu}\frac{I^i - p^a_t\tilde{c}_a}{p^m_t} \\ c^a_{i,t,t+1} &= \frac{\delta\theta}{1-\theta+\delta+\nu}\frac{(I^i - p^a_t\tilde{c}_a)r_{K,t+1}}{p^a_{t+1}p^m_t} \\ c^m_{i,t,t+1} &= \frac{\delta(1-\theta)}{1-\theta+\delta+\nu}\frac{(I^i - p^a_t\tilde{c}_a)r_{K,t+1}}{p^m_{t+1}p^m_t} \\ n^u_{i,t} + \gamma^s n^s_{i,t} &= \frac{\nu}{1-\theta+\delta+\nu}\frac{I^i - p^a_t\tilde{c}_a}{w^i_t\tau + p^a_t\epsilon_a\tilde{c}_a} \end{cases}$$

with :

$$s^i_t = \frac{\delta}{1-\theta+\delta+\nu}\frac{I^i - p^a_t\tilde{c}_a}{p^m_t}$$

and :

$$l^i_t = 1 - \tau\frac{\nu}{1-\theta+\delta+\nu}\frac{I^i - p^a_t\tilde{c}_a}{w^i_t\tau + p^a_t\epsilon_a\tilde{c}_a}$$

# Budget allocation

If the subsistence constraint is not binding (i.e. $\frac{\theta}{1+\delta+\nu}\frac{I}{p^a_t} \geq \tilde{c}_a$) :

$$\begin{cases} c^a_{i,t,t} & = \frac{\theta}{1+\delta+\nu}\frac{I^i}{p^a_t} \\ c^m_{i,t,t} & = \frac{1-\theta}{1+\delta+\nu}\frac{I^i}{p^m_t} \\ c^a_{i,t,t+1} & = \frac{\delta\theta}{1+\delta+\nu}\frac{I^i r_{K,t+1}}{p^a_{t+1}p^m_t} \\ c^m_{i,t,t+1} & = \frac{\delta(1-\theta)}{1+\delta+\nu}\frac{I^i r_{K,t+1}}{p^m_{t+1}p^m_t} \\ n^u_{i,t} + \gamma^s n^s_{i,t} & = \frac{\nu}{1+\delta+\nu}\frac{I^i}{w^i_t\tau + p^a_t \epsilon_a \tilde{c}_a} \end{cases}$$

with :

$$s^i_t = \frac{\delta}{1+\delta+\nu}\frac{I^i}{p^m_t}$$

and :

$$l^i_t = 1 - \tau\frac{\nu}{1+\delta+\nu}\frac{I^i}{w^i_t\tau + p^a_t\epsilon_a\tilde{c}_a}$$

# Total population and population dynamics

Thus, we define a "total unskilled children aggregate"
$M_{t+1} = N_t^u(n_{u,t}^u + \gamma^s n_{u,t}^s) + N_t^s(n_{u,t}^u + \gamma^s n_{u,t}^s) = N_{t+1}^u + \gamma N_{t+1}^s$,
which we can use to determine the qualified/unqualified distribution by resolving the general equilibrium of the following period (we suppose that agents at time $t$ have a perfect foresight of the situation at time $t+1$), with the condition : if $N_{t+1}^s \neq 0$ then $\frac{w_{t+1}^s}{w_{t+1}^u} = \gamma$.
Interestingly, with this aggregate, whatever the distribution is between skilled and unskilled at $t+1$, the consumption of food and the child-rearing time are unaffected.
At time $t$, the total population is given by :

$$Population_t = \frac{1}{2}(N_{t+1}^u + N_{t+1}^s) + N_t^u + N_t^s + N_{t-1}^u + N_{t-1}^s$$

# Outline

# Overview

- We consider four different energy sources with their own characteristics : traditional biomass, coal, hydrocarbons (oil and gaz), and renewables.
- It is assumed that the only factor of production of energy is capital (except for traditional biomass, for which we also consider traditional biomass).
- Considering a fixed extraction quantity, technology lowers costs.
- For non-renewable energy, the decrease of the stock implies an increase of the cost of extraction.
- We assume that representative firms maximise profit and face a zero-profit condition (perfect competition)

# Overview

- Traditional biomass : corresponds to firewood which was the main source of energy worldwide until the industrial revolutions (and is still a major source of energy worldwide). Its production is relatively inefficient and is limited by the surface area allocated to it (Thus creating a competition with agriculture, which encourages the use of other forms of energy such as coal).
- Coal : it is the easiest technology to master, coal has poor calorific capacity (and therefore low thermodynamic efficiency and numerous greenhouse gas emissions), the global stock is quite large.
- Oil and gas : More complex technology, better but less stock
- Renewable energy (wind, hydro, nuclear, solar, "modern" biomass) : Fairly complex technology, low efficiency but (nearly) unlimited stock

# Traditional biomass

Biomass is a renewable energy since it has no stocks. It is assumed to have the following production function :

$$E^{bio}(K,X) = A^{bio} K^{\alpha^{bio}} X^{1-\alpha^{bio}}$$

Biomass production requires some capital and depends mainly on the land, thus creating a balance with the surface area allocated to agriculture.
Progress is globally non-existent : it is assumed that trees do not grow faster with knowledge and technology.

# Traditional biomass

Thus, the demands of production factors are

$$K_t^{bio} = \frac{\alpha p_t^{E^{bio}}}{r_{K,t}} E_t^{bio}$$

$$X_t^{bio} = \frac{(1-\alpha) p_t^{E^{bio}}}{r_{L,t}} E_t^{bio}$$

And the price is given by :

$$p_t^{E^{bio}} = \frac{1}{A^{bio}} \frac{r_{K,t}^{\alpha^{bio}} r_{L,t}^{1-\alpha^{bio}}}{(\alpha^{bio})^{\alpha^{bio}} (1-\alpha^{bio})^{1-\alpha^{bio}}}$$

# Coal

Each extraction reduces the stock $R$. Since an additional extraction is harder (it is necessary to go deeper to find new fields), the extraction cost decreases with the stock.
In addition, technical progress has an effect on the extraction of energy by lowering costs.
The following extraction function is therefore proposed :

$$E_t^{coal}(K) = f^{coal}(Q_t)\left(\frac{R_t^{coal}}{R_0^{coal}}\right)^{\sigma^{coal}} K \text{ if } f^{coal}(Q_t) > 0$$

with :

$$R_{t+1}^{coal} = R_t^{coal} - E_t^{coal}$$

# Coal

Thus :

$$K_t^{coal} = \frac{1}{f^{coal}(Q_t)\left(\frac{R_t^{coal}}{R_0^{coal}}\right)^{\sigma^{coal}}} E_t^{coal} \text{ if } f^{coal}(Q_t) > 0$$

and :

$$p_t^{E^{coal}} = \frac{r_{K,t}}{f^{coal}(Q_t)\left(\frac{R_t^{coal}}{R_0^{coal}}\right)^{\sigma^{coal}}} \text{ if } f^{coal}(Q_t) > 0$$

# Hydrocarbons

Just like coal, we assume :

$$E_t^{hydro}(K) = f^{hydro}(Q_t)\left(\frac{R_t^{hydro}}{R_0^{hydro}}\right)^{\sigma^{hydro}} K \text{ if } f^{oil}(Q_t) > 0$$

with :

$$R_{t+1}^{hydro} = R_t^{hydro} - E_t^{hydro}$$

So :

$$K_t^{hydro} = \frac{1}{f^{hydro}(Q_t)\left(\frac{R_t^{hydro}}{R_0^{hydro}}\right)^{\sigma^{hydro}}} E_t^{hydro} \text{ if } f^{oil}(Q_t) > 0$$

and :

$$p_t^{E^{hydro}} = \frac{r_{K,t}}{f^{hydro}(Q_t)\left(\frac{R_t^{hydro}}{R_0^{hydro}}\right)^{\sigma^{hydro}}} \text{ if } f^{oil}(Q_t) > 0$$

# Renewables

As with other technologies, the cost of production is diminishing with technical progress.
Finally, a renewable energy is like a non-renewable energy with a stock that regenerates at each period. Hence the production function :

$$E_t^{renew}(K) = min(f^{renew}(Q_t)K, E_{max}^{renew}) \text{ if } f^{renew}(Q_t) > 0$$

So :

$$K_t^{renew} = \frac{1}{f^{renew}(Q_t)} E_t^{renew} \text{ if } f^{renew}(Q_t) > 0$$

and :

$$p_t^{E^{renew}} = \frac{r_{K,t}}{f^{hydro}(Q_t)} \text{ if } f^{renew}(Q_t) > 0$$

# Outline

# Overview of consumption goods production

- In order to highlight the fundamental role of agriculture in economic development ([Federico, 2015], [Federico, 2004], [Strulik and Weisdorf, 2008]), we work with two productive sectors, an agricultural sector and a manufacturing sector.
- Moreover, to present the complementarity between capital-labour and energy ([Smil, 2018]), we model the production functions with nested-CES functions of the KL and E aggregates ([Stern and Kander, 2012]).
- Finally, in order to represent the two growth regimes, we assume the existence of two productive modes of the manufactured good, the first one, denoted old technology, that does not require the use of skilled labour and the other one, denoted new technology, that does, like in [Galor and Mountford, 2008].

# Agricultural sector : production

We set the following production function :

$$F^a(K, L^u, E^i, T) = (1 - D^a(T)) \left( (A^a_{LLK} LLK)^{\frac{\eta^a - 1}{\eta^a}} + (A^a_E E^a)^{\frac{\eta^a - 1}{\eta^a}} \right)^{\frac{\eta^a}{\eta^a - 1}}$$

where $LLK$ is a land, labor and capital aggregate :

$$LLK(K, L^u, X) = \left( {\alpha^a_{LLK}}^{\frac{1}{\eta^a_{LLK}}} K^{\frac{\eta^a_{LLK} - 1}{\eta^a_{LLK}}} + {\beta^a_{LLK}}^{\frac{1}{\eta^a_{LLK}}} X^{\frac{\eta^a_{LLK} - 1}{\eta^a_{LLK}}} + (1 - \alpha^a_{LLK} - \beta^a_{LLK})^{\frac{1}{\eta^a_{LLK}}} {L^u}^{\frac{\eta^a_{LLK} - 1}{\eta^a_{LLK}}} \right)^{\frac{\eta^a_{LLK}}{\eta^a_{LLK} - 1}}$$

# Agricultural sector : production

And $E$ is an aggregate of the different energy sources :

$$E^a = \left( \sum_i (\rho^i E^i)^{\frac{\eta_E^a - 1}{\eta_E^a}} \right)^{\frac{\eta_E^a}{\eta_E^a - 1}}$$

$A^a_{LLK}$ denotes the TFP of the agricultural sector (increasing function of $Q_t$) and, $A^a_E$ denotes the productivity term for energy.

# Agricultural sector : prices

Prices of aggregates are :

$$p^{LLK} = \Big(\alpha^a_{LLK} r_K^{1-\eta^a_{LLK}} + \beta^a_{LLK} r_L^{1-\eta^a_{LLK}} + (1 - \alpha^a_{LLK} - \beta^a_{LLK}) w^{u\,1-\eta^a_{LLK}}\Big)^{\frac{1}{1-\eta^a_{LLK}}}$$

$$p^{E^a} = \left(\sum_i \left(\frac{p^{E^i}}{\rho^i}\right)^{1-\eta^a_E}\right)^{\frac{1}{1-\eta^a_E}}$$

And thus the price of the agricultural good :

$$p^a = \frac{1}{1 - D^a(T)} \left(\left(\frac{p^{LLK}}{A^a_{LLK}}\right)^{1-\eta^a} + \left(\frac{p^{E^a}}{A^a_E}\right)^{1-\eta^a}\right)^{\frac{1}{1-\eta^a}}$$

# Agricultural sector : demand

We obtain the following demand of production factors and energy :

$$
\begin{cases}
K^a &= \alpha^a_{LLK} \left(\frac{r_K}{p^{LLK}}\right)^{-\eta^a_{LLK}} \left(A^a_{LLK}(1-D^a(T))\right)^{\eta^a-1} \left(\frac{p_{LLK}}{p^a}\right)^{-\eta^a} Y^a \\
X^a &= \beta^a_{LLK} \left(\frac{r_L}{p^{LLK}}\right)^{-\eta^a_{LLk}} \left(A^a_{LLK}(1-D^a(T))\right)^{\eta^a-1} \left(\frac{p_{LLK}}{p^a}\right)^{-\eta^a} Y^a \\
L^{u,a} &= (1-\alpha^a_{LLK}-\beta^a_{LLK}) \left(\frac{w^u}{p^{LLK}}\right)^{-\eta^a_{LLk}} \\
& \qquad \left(A^a_{LLK}(1-D^a(T))\right)^{\eta^a-1} \left(\frac{p_{LLK}}{p^a}\right)^{-\eta^a} Y^a \\
E^{a,i} &= \frac{1}{\rho^i} \left(\frac{p^{E^i}}{\rho^i p^{E^a}}\right)^{-\eta^a_E} \left(A^a_E(1-D^a(T))\right)^{\eta^a-1} \left(\frac{p^{E^a}}{p^a}\right)^{-\eta^a} Y^a
\end{cases}
$$

# Manufacturing sector - Old technology : production and price

Concerning the old technology, we consider the following production function :

$$F^{m,old}(L^u, E^{bio}, T) = (1 - D^m(T)) \left( (A_L^{m,old} L^u)^{\frac{\eta^{m,old}-1}{\eta^{m,old}}} + (A_E^{m,old} \rho^{bio} E^{bio})^{\frac{\eta^{m,old}-1}{\eta^{m,old}}} \right)^{\frac{\eta^{m,old}}{\eta^{m,old}-1}}$$

So the price of the good produced with the old technology is :

$$p^{m,old} = \frac{1}{1-D^m(T)} \left( \left( \frac{w^u}{A_L^{m,old}} \right)^{1-\eta^{m,old}} + \left( \frac{p^{E^{bio}}}{A_E^{m,old} \rho^{bio}} \right)^{1-\eta^{m,old}} \right)^{\frac{1}{1-\eta^{m,old}}}$$

# Manufacturing sector - Old technology : demand

The demand for both factors of production is :

$$\begin{cases} L^{u,a} &= (A_L^{m,old}(1-D^m(T))^{\eta^{m,old}-1}\left(\frac{w^u}{p^{m,old}}\right)^{-\eta^{m,old}} Y^{m,old} \\ E^{m,old,bio} &= \frac{1}{\rho^b io}(A_E^{m,old}(1-D^m(T))^{\eta^{m,old}-1} \\ & \qquad \left(\frac{p^{E^{bio}}}{\rho^{bio} p^{m,old}}\right)^{-\eta^{m,old}} Y^{m,old} \end{cases}$$

# Manufacturing sector - Modern technology : production

With the new technology, the production function takes the form

$$F^{m,new}(K, L^u, L^s, E^i, T) = (1 - D^m(T)) \left( (A_{KL}^{m,new} KL)^{\frac{\eta^{m,new}-1}{\eta^{m,new}}} + (A_E^{m,new} E^{m,new})^{\frac{\eta^{m,new}-1}{\eta^{m,new}}} \right)^{\frac{\eta^{m,new}}{\eta^{m,new}-1}}$$

Where :

$$KL = \left( \alpha_{KL}^{m,new\,\frac{1}{\eta_{KL}^{m,new}}} K^{\frac{\eta_{KL}^{m,new}-1}{\eta_{KL}^{m,new}}} + (1 - \alpha_{KL}^{m,new})^{\frac{1}{\eta_{KL}^{m,new}}} L^{m,new\,\frac{\eta_{KL}^{m,new}-1}{\eta_{KL}^{m,new}}} \right)^{\frac{\eta_{KL}^{m,new}}{\eta_{KL}^{m,new}-1}}$$

# Manufacturing sector - Modern technology : production

With :

$$L^{m,new} = \left( \alpha_L^{m,new\,\frac{1}{\eta_L^{m,new}}} (L^s)^{\frac{\eta_L^{m,new}-1}{\eta_L^{m,new}}} + (1-\alpha_L^{m,new})^{\frac{1}{\eta_L^{m,new}}} (L^u)^{\frac{\eta_L^{m,new}-1}{\eta_L^{m,new}}} \right)^{\frac{\eta_L^{m,new}}{\eta_L^{m,new}-1}}$$

And, finally :

$$E^{m,new} = \left( \sum_i (\rho^i E^{m,new,i})^{\frac{\eta_E^{m,new}-1}{\eta_E^{m,new}}} \right)^{\frac{\eta_E^{m,new}}{\eta_E^{m,new}-1}}$$

# Manufacturing sector - Modern technology : prices

The prices of aggregates are :

$$W = \left(\alpha_L^{m,new} {w^s}^{1-\eta_L^{m,new}} + (1-\alpha_L^{m,new}) {w^u}^{1-\eta_L^{m,new}}\right)^{\frac{1}{1-\eta_L^{m,new}}}$$

$$p^{KL} = \left(\alpha_{KL}^{m,new} r_K^{1-\eta_{KL}^{m,new}} + (1-\alpha_{KL}^{m,new}) W^{1-\eta_{KL}^{m,new}}\right)^{\frac{1}{1-\eta_{KL}^{m,new}}}$$

$$p^{E^{m,new}} = \left(\sum_i \left(\frac{p^{E^i}}{\rho^i}\right)^{1-\eta_E^{m,new}}\right)^{\frac{1}{1-\eta_E^{m,new}}}$$

And thus the price of the agricultural good :

$$p^{m,new} = \frac{1}{1-D^m(T)} \left(\left(\frac{p^{KL}}{A_{KL}^{m,new}}\right)^{1-\eta^{m,new}} + \left(\frac{p^{E^{m,new}}}{A_E^{m,new}}\right)^{1-\eta^{m,new}}\right)^{\frac{1}{1-\eta^{m,new}}}$$

# Manufacturing sector - Modern technology : prices

The demand for production factors is :

$$
\begin{cases}
K^{m,new} = \alpha_{LLK}^{m,new} \left(\frac{r_K}{p^{KL}}\right)^{-\eta_{KL}^{m,new}} \left(A_{KL}^{m,new}(1-D^m(T))\right)^{\eta^{m,new}-1} \left(\frac{p_{KL}}{p^{m,new}}\right)^{-\eta^{m,new}} \\
L^{s,m,new} = \alpha_{L}^{m,new} \left(\frac{w^s}{W}\right)^{-\eta_{L}^{m,new}} \left(1-\alpha_{KL}^{m,new}\right) \left(\frac{W}{p^{KL}}\right)^{-\eta_{LLk}^{m,new}} \\
\qquad \left(A_{KL}^{m,new}(1-D^m(T))\right)^{\eta^{m,new}-1} \left(\frac{p_{KL}}{p^{m,new}}\right)^{-\eta^{m,new}} Y^{m,new} \\
L^{u,m,new} = \left(1-\alpha_{L}^{m,new}\right) \left(\frac{w^u}{W}\right)^{-\eta_{L}^{m,new}} \left(1-\alpha_{KL}^{m,new}\right) \left(\frac{W}{p^{KL}}\right)^{-\eta_{LLk}^{m,new}} \\
\qquad \left(A_{KL}^{m,new}(1-D^m(T))\right)^{\eta^{m,new}-1} \left(\frac{p_{KL}}{p^{m,new}}\right)^{-\eta^{m,new}} Y^{m,new} \\
E^{a,i} = \frac{1}{\rho^i} \left(\frac{p^{E^i}}{\rho^i p^{E m,new}}\right)^{-\eta_{E}^{m,new}} \left(A_{E}^{m,new}(1-D^m(T))\right)^{\eta^{m,new}-1} \left(\frac{p^{E m,new}}{p^{m,new}}\right)^{-\eta^{m,n}}
\end{cases}
$$

# Outline

# Carbon cycle

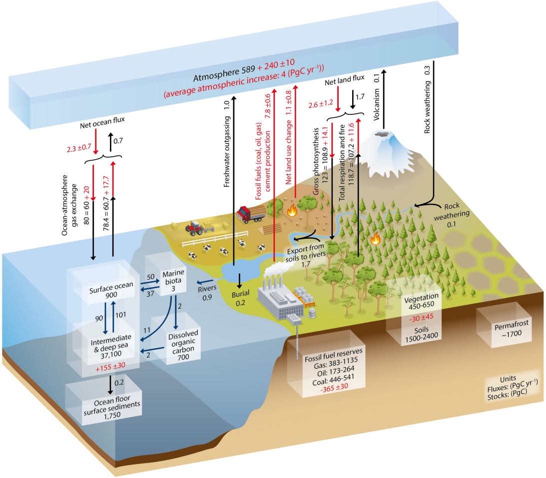


Figure – Carbon cycle dynamics - Source : [IPCC, 2014]

# Carbon dioxyde emissions and carbon cycle

Carbon dioxyde emissions are given by :

$$CO_{2,t} = \xi^{coal} E_t^{coal} + \xi^{hydro} E_t^{hydro} + CO_{2,t}^{ext}$$

With $\xi^i$ the carbon intensity of an energy. We consider the linear 3-box model used in [Nordhaus, 2017] :

$$\begin{cases} m_{t+1}^{atm} = & c_{atm \longrightarrow atm} m_t^{atm} + c_{up \longrightarrow atm} m_t^{up} + CO_{2,t} \\ m_{t+1}^{up} = & c_{atm \longrightarrow up} m_t^{atm} + c_{up \longrightarrow up} m_t^{up} + c_{deep \longrightarrow up} m_t^{deep} \\ m_{t+1}^{deep} = & c_{up \longrightarrow deep} m_t^{up} + c_{deep \longrightarrow deep} m_t^{deep} \end{cases}$$

It allows us to derive $m_t^{atm}$, the concentration of $CO_2$ in the atmosphere at each period.

# Radiative forcing and global warming

With the concentration of $CO_2$ in the atmosphere defined, it is possible to derive the radiative forcing :

$$RF_{t+1} = \frac{F_{2\times CO_2}}{log(2)} log\left(\frac{m^{atm}_{t+1}}{m^{atm}_{eq}}\right) + RF^{ext}_{t+1}$$

And then the dynamics of global mean surface temperature and mean ocean temperature are given by :

$$T^{atm}_{t+1} = T^{atm}_t + c_1(RF_{t+1} - \frac{F_{2\times CO_2}}{T^{atm}_{2\times CO_2}} T^{atm}_t) - c_1 c_3(T^{atm}_t - T^{ocean}_t)$$

$$T^{ocean}_{t+1} = T^{ocean}_t + c_4(T^{atm}_t - T^{ocean}_t)$$

# Climate damages

Concerning climate damages, we consider differentiated damages depending on the sector (agriculture is considered as more vulnerable to climate change than industry) that are taken from [Desmet and Rossi-Hansberg, 2015] :

$$D^a(T) = d_1^a T + d_2^a T^2$$

$$D^m(T) = d_1^m T + d_2^m T^2$$

# Climate damages

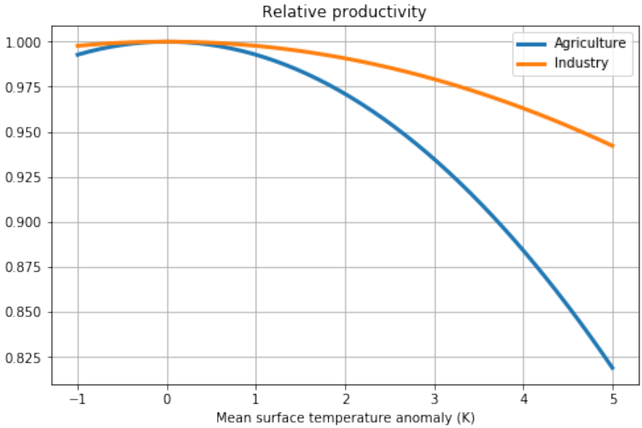


Figure – Variation of productivity due to climate change - Source : [Desmet and Rossi-Hansberg, 2015]

# Outline

# Population dynamics

At time $t$, the total population is given by :

$$Population_t = \frac{1}{2}(N^u_{t+1} + N^s_{t+1}) + N^u_t + N^s_t + N^u_{t-1} + N^s_{t-1}$$

There is a regime shift (shift from the old technology to the modern one, and a rise in skilled labor) if, when agents consider their expectation of the next period we observe :

$$\frac{w^u}{p^m}\Big|_{Old\ technology} < \frac{w^u}{p^m}\Big|_{Modern\ technology}$$

i.e.

$$\frac{\partial F^{m,old}}{\partial L^u}(L^{u*}, E^{bio*}, T) < \frac{\partial F^{m,new}}{\partial L^u}(K^*, L^{u*}, L^{s*}, E^{i*}, T)$$

# Common knowledge index

Thus it is possible to define $h_t = \frac{N_t^s}{N_t^s + N_t^u}$ the share of skilled agents in the population at time $t$ and $Q_t$, a synthetic indicator of progress :

$$\frac{Q_{t+1} - Q_t}{Q_t} = g(h_t) \text{ i.e. } Q_{t+1} = (1 + g(h_t))Q_t$$

where $g$ is such that $g(0) > 0$, $g' > 0$ and $g'' < 0$.
$Q_t$ has various signification :

- a human capital index
- a technology index
- a form of useful knowledge ([Strulik et al., 2013])
- a GPT driver index ([Schaefer et al., 2014])

# Extraction progress

We model the ability to extract some materials with sigmoids $f^i(Q_t)$ that are driven by $Q_t$ :

$$f^i(Q_t) = A^i_{lim} tanh(scale^i(Q_t - Q^i_{min}))$$

Two ideas :

- The concept of emergence of a technology
- the extracting yield is bounded and that it converges towards its limit with technological development

# Outline

# Total population dynamics

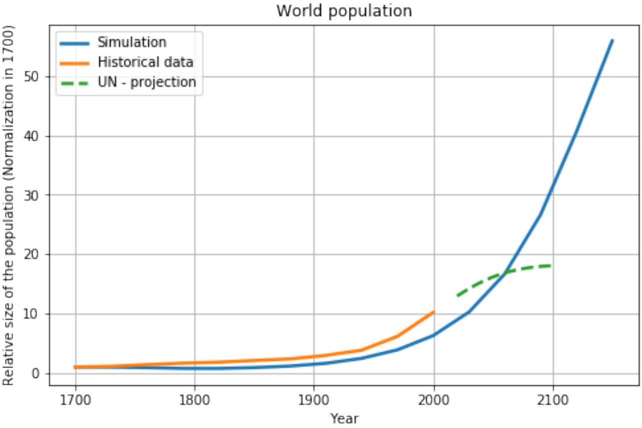


Figure – Population simulation, the population in 1700 is normalized to 1 - Sources : Hyde database [Klein Goldewijk et al., 2011], UN World Population Prospects 2019

# Population growth rate


Figure – Population growth rate simulation - Sources : Hyde database [Klein Goldewijk et al., 2011], UN World Population Prospects 2019

# GDP per capita

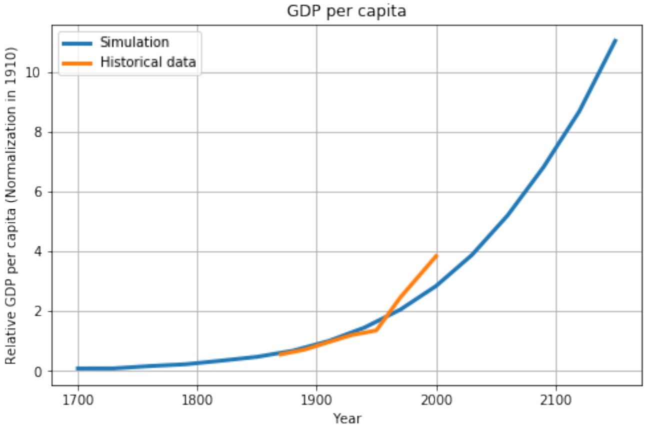


Figure – GDP per capita, the GDP per capita in 1910 is normalized to 1 - Source : Maddisson Project Database 2018 [Bolt et al., 2018]

# Global energy transition

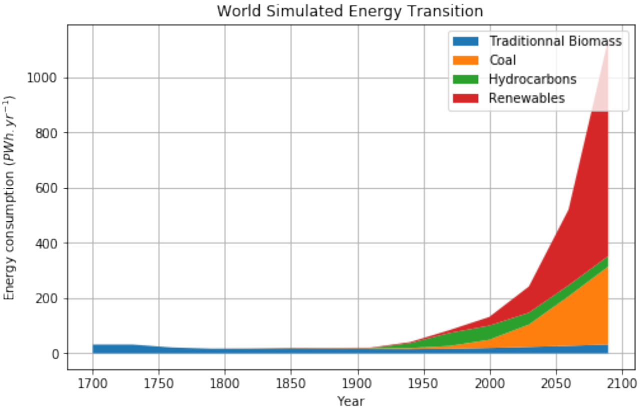


Figure – Energy transition simulation

# Coal extraction

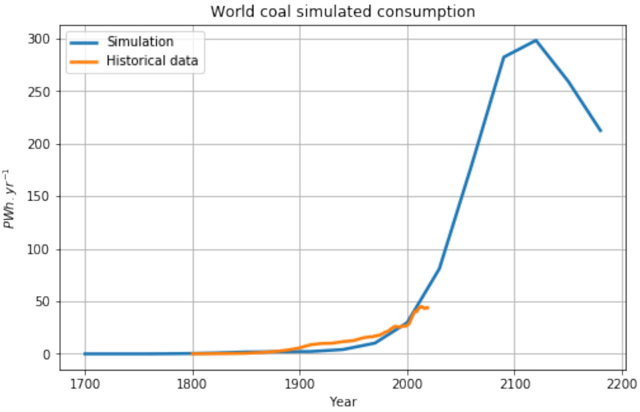


Figure – Coal consumption simulation - Source : [Smil, 2016], [British Petroleum, 2019]

# Hydrocarbons extraction

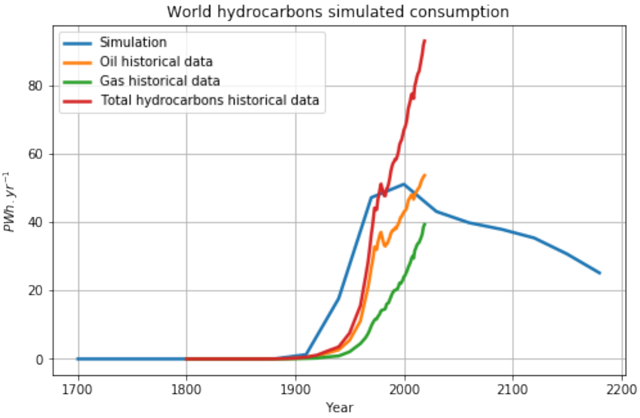


Figure – Hydrocarbon consumption simulation - Source : [Smil, 2016], [British Petroleum, 2019]

# GMST anomaly

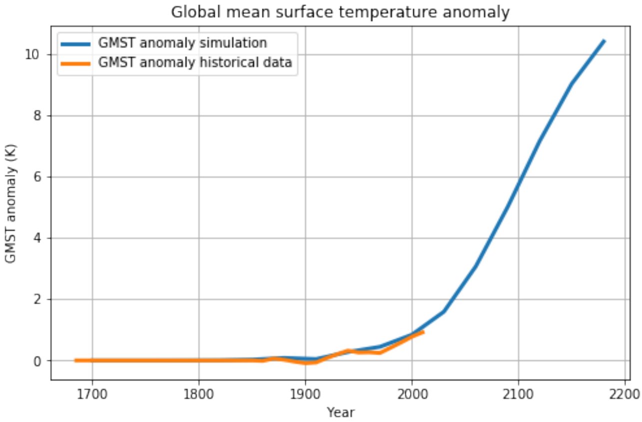


Figure – Global mean surface temperature anomaly - Source : [IPCC, 2014]

# Outline

# A matter of disaggregation

- The interest of studying an endogenous demography is to see the "right" economic-demographic coupling appear.
- However, this coupling is not evenly distributed over time throughout the world : some countries have long since completed their demographic transition, while others are still in its first phase.
- This has for consequence that with too much aggregation the information given by the demographic, income and performance data is diluted, which makes it complex to obtain a range of quantitative results of good quality.
- Moreover, it seems above all more interesting in economic terms to distinguish between regions that differ in their economic and demographic trajectories in order to consider population and future development in the climate change responsibility debate.

# Estimation of climate damages

- The estimation of climate change impacts is a heated debate.
- Indeed, it seems difficult to consider econometric estimates alone as capable of determining the damage of global warming.
- The right logic would be to succeed in considering the damage via a structural modelling of the impacts of global warming. However, the link between these impacts on, for example, ecosystem services and the economy or the well-being of agents is not at all obvious to determine.
- Two elements of solutions : On the one hand, it is necessary to explore several types of damage and to consider the uncertainty about the said damages and, on the other hand, to develop hybrid methods for estimating this damage in a disaggregated manner ([Hsiang et al., 2017], [Burke et al., 2015]).

# Techno-economic structure and bounds of energy production

- The results we are obtaining give us a huge increase in the use of renewable energies : the energy available per person is not really in contraction.
- However, as for non-renewable energy, we can expect the production cost of renewable energy to increase with the installed production capacity (considering the constant technological level), because energy has to be extracted from less accessible fields, where availability is less important ...
- Thus, a big reflection is needed both on the model used concerning renewable energies (because it is for the moment completely linear) and on its calibration !

# Governmental action and change in agents behaviour

- Our model is not normative, but positive : it does not try to find the optimal trajectory but the most probable trajectory given the agents' preferences.
- However, it seems possible and valuable to model behavioural changes that represent both changes in socio-cultural values and changes in the way the environment is taken into account.
- Two ideas for translating this : Governmental action of taxation or regulation on the number of children or on emissions (change in the budget and constraints of agents or companies), change in the preferences of agents (variation in the parameters of the utility function).

# Skill-biased technological progress

- The need of two different technologies in the manufacturing sector is a modelling need to represent a skill bias in the technology development.
- Indeed, unskilled labor as the only input in the old technology is equivalent to an elasticity of substitution equal to $+\infty$ with our framework
- The idea would then be that the elasticity of substitution varies with time from very high to a lower value (which is not in contradiction with the long-term decrease of the skill premium, since it depends as well on the composition of the labor market)
- Then, the skill premium should not be fixed and the utility function should be different (maybe like in [Burzynnskia et al., 2019])

# Outline

# Conclusion

- For a stylized model, it's not so bad !
- A work which appears very preliminary and incomplete, I thus retain the qualifier of "promising".

# Conclusion

- Personally, I am very happy to have been able to carry out this work, which I would describe as quite ambitious.
- However, it is clearly not finished, what I presented to you corresponds to a big roughing, we must continue and move forward !
- Finally, I would like to thank you, Katheline and Hippolyte for your support and guidance.

# Conclusions

Thank you for your attention, any question ?

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